MATH2017 Homework 3 Feedback

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Question 1

1. This is just the definition - you need to justify why it holds.

2. You have the correct idea of how to prove α is a supremum (or infimum) of A, but you need to be careful with the details.

3. Note that upper (resp lower) bounds are not unique. A set which has an upper bound has infinitely many upper bounds (resp lower bounds) (eg if α is UB for A, then so is $\alpha + \epsilon$ for any $\epsilon > 0$).

4. Some used the fact that $\sup(A) + \sup(B) = \sup(A + B)$ and split the set into a sequence and the open interval $(0, 1)$. This is acceptable if you either prove it or have done it in lectures, but it means you have twice as many sup's and inf's to prove.

5. Note that $1 \notin (0, 1)$. Similarly, $0 \notin (0, 1)$. If you assume either of those two, the proof for the sup/inf becomes much easier.

6. Note that 1 is not an upper bound on $\frac{1}{n}$, as the latter is not a set. 1 is an upper bound for $\{\frac{1}{n} : n \in \mathbb{Z}^+\}$. Similarly for 0 being a lower bound. Similarly, x is not a set, so you can't take $\sup(x)$.

7. Note that an element in A has the form $x + 1/n$ for some $x \in (0, 1)$ and $n \in \mathbb{Z}^+$. Thus, to say that some number a is an element of A, you need to express it as $x + 1/n$, not just say $a \in (0, 1)$ or something similar (as '0' is not part of the sequence $(\frac{1}{n})$).

   Question 2

8. Once again, make sure you give precise values, not some approximations. In particular, if you just gave a numeric approximation to the upper and lower Reimann sums, and have some of the other comments on this question, you have lost a mark for that, overall.

9. Simplify your expression.

10. It would be good to note when log is increasing/decreasing, hence justify why the sample points you chose are the correct ones for upper/lower Riemann sums. For those that drew diagrams, well done! Adding some sentences of explanation would be good, too.

    Question 3

11. You should mention which step follows from your inductive hypothesis for part a). Similarly, in the solution to part b), mention where you use the result from part a) and where you use Theorem 90.

12. When doing proofs by induction, don't forget to write that. Some have lost a point for poor exposition - it's not that long a proof, so at least write it properly.

13. Remember that when using a theorem, you need to check all conditions are satisfied (eg - bounded, increasing/decreasing, etc).

14. General remark: when I write 'good exposition' or 'well-written', this means that the mathematics is well-written, carefully justified, and presented well. It's not a comment on your handwriting.

    Question 4

15. Note that you need to choose a special, 'useful', dissection of the interval $[0, 2]$. Any will 'work', but it's better to cut down on calculations.

16. Note that if $A \subset B$, then $\inf(A) \geqslant \inf(B)$ and $\sup(A) \leqslant \sup(B)$.

17. Overall Q4 was well done!

    Question 5

18. Overall well done! A few students opted for more creative solutions, but they were still (mostly) correct. For those that did this, go through and check the individual comments made.